## IAPMO/ASPE WHITE PAPER 2-2024

## Capacities of Stacks and Horizontal Drains in Storm Drainage Systems

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## Contents

Foreword and Purpose ..... ii
Introduction .....  1
Greek Symbols .....  1
Nomenclature .....  2
1.0 Materials .....  3
2.0 Flow Capacity Equations .....  6
2.1 Vertical Storm Drain Sizing .....  6
2.2 Horizontal Storm Drain Sizing - Manning ..... 10
2.3 Horizontal Storm Drain Sizing - Darcy-Weisbach ..... 14
3.0 Discussion ..... 19
4.0 Conclusion ..... 22
5.0 References ..... 23

## Foreword and Purpose

The sizing for storm drainage systems are dependent upon flow capacity equations used to calculate velocities and flow rates in pipe conduits. One of the variables in the equations is the coefficient of roughness. This paper explores how the roughness of different types of material will change the computational results that are dependent upon the roughness coefficient of the pipe.

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# IAPMO/ASPE White Paper Capacities of Stacks and Horizontal Drains in Storm Drainage Systems 

## Introduction

The sizing for storm drainage systems has been committed to plumbing codes with mandatory sizing tables for engineering design. Although the plumbing codes allow for deviation based on recognized engineering practices and equivalency of effectiveness, the designs of storm drainage systems generally default to the plumbing codes. The sizing tables in the U.S. plumbing codes stem from either the ASA A40.8 National Plumbing Code from 1955 (ASME, 1955) or the computation from the National Bureau of Standards (NBS) Monograph 31 (Wyly, Eaton, 1961). It is probable that the National Plumbing Code was influenced by the research of Dawson and Kalinske, University of lowa, published in Bulletin 10 that referenced the Manning equation (Dawson, Kalinske, 1937).

The original research of the NBS and the University of lowa dated from the time when cast iron was the common pipe material for both the sanitary and storm drainage systems. Since then, plastics have emerged as a dominant material for plumbing systems. Therefore Section 1.0 begins with identifying the allowable types of material for storm drainage systems in three plumbing codes used in the US. This study considers how the varying roughness of the different types of material will change the computational results that are dependent upon the roughness coefficient of the pipe.

Section 2.0 examines the equations used to compute capacities for vertical and horizontal drains and assesses the outcomes. Better understanding of the equations will facilitate decision-making for sizing calculations. Three equations are introduced in this study, one for vertical drain capacity and two for horizontal drain capacity. The roughness coefficient for each equation is considered for two types of pipe material, PVC and cast iron, showing comparative results. Annular ratios for vertical stacks and flow depth for horizontal drains are also examined to consider the effect on pipe capacity.

Section 3.0 discusses the comparative differences in calculating flow rates when changing the roughness coefficient for two types of material. The results of this analysis and evaluation demonstrate the need to consider the roughness of pipe material when sizing storm drainage systems.

## Greek Symbols

$\varepsilon=$ absolute roughness, feet
$\theta=$ radians computed as $2 \cos ^{-1} \frac{r-h}{r}$

## Nomenclature

$\mathrm{A}=$ cross sectional area of pipe, $\mathrm{ft}^{2} \frac{\pi D^{2}}{4}$
$\mathrm{A}_{\mathrm{an}}=$ annular cross-sectional area of pipe, $\mathrm{ft}^{2} \frac{\pi r_{s} D^{2}}{4}$
$\mathrm{a}=$ the time rate of roughness increase, $\mathrm{ft} / \mathrm{yr}$
$\mathrm{C}=$ constant, dimensionless
$\mathrm{c}=$ coefficient, dimensionless
$\mathrm{D}=$ diameter of pipe, feet
$d=$ diameter of pipe, inches
$f=$ friction coefficient, dimensionless
$\mathrm{g}=$ acceleration due to gravity ( $32.2 \mathrm{ft} / \mathrm{s} / \mathrm{s}$ )
$\mathrm{h}=$ depth flow, ft
$\mathrm{h}_{f}=$ pressure loss over distance L
$\mathrm{k}=$ roughness of pipe at any time $\mathrm{t}, \mathrm{ft}$
$\mathrm{k}_{0}=$ roughness of new material, ft
$\mathrm{L}=$ length of pipe, ft
$\mathrm{n}=$ Manning's coefficient for roughness
$\mathrm{P}=$ wetted perimeter, ft computed as $\mathrm{r}^{*} 2 \cos ^{-1} \frac{r-h}{r}$
$\mathrm{Q}=$ volume rate of flow, cf/s
$q=$ volume rate of flow, gpm
Re $=$ Reynolds number, dimensionless
$\mathrm{R}_{\mathrm{h}}=$ hydraulic radius, ft computed as $\mathrm{R}=\frac{A}{P}=\frac{\frac{r^{2}(\theta-\sin \theta)}{2}}{r \theta}$
$r=$ radius, ft
$r_{h}=$ hydraulic radius, $m$
$r_{s}=$ ratio of area of cross section of water stream in a drainage stack to total area of cross section of the stack
$\mathrm{S}=$ slope, $\mathrm{ft} / \mathrm{ft}$
$\mathrm{s}=$ slope, $\mathrm{m} / \mathrm{m}$
$t=$ time interval, yr
$\mathrm{V}=$ velocity, $\mathrm{ft} / \mathrm{s}$
$v=$ velocity, $\mathrm{m} / \mathrm{s}$
$\mathrm{V}_{t}=$ mean terminal velocity of water flowing on the wall of the drainage stack, $\mathrm{ft} / \mathrm{s}$

### 1.0 Materials

Materials used for storm drainage systems were selected from the 2024 IAPMO Uniform Plumbing Code (UPC), 2024 International Plumbing Code (IPC), and the 2024 National Standard Plumbing Code (NSPC). The applicable materials for the purpose of this report were selected for vertical conductors and horizontal building storm drains and building storm sewers.

Table 1.0 compiles these materials into a single table. The $x$ 's in the columns indicate the approval of the material for the use designated in the header. Gutters and downspouts that are sheet metal of galvanized steel, aluminum, or copper are omitted from this report. The last two columns on the right in Table 1.0 include coefficients for absolute roughness ( $\varepsilon$ ) and for Manning's coefficient ( $n$ ) corresponding to the type of material. Where a material is not approved for either a vertical stack or horizontal drain, the applicable coefficient is not provided. These coefficients will be needed when using the pipe sizing equations presented in this paper. The flow capacity will vary where the coefficients differ between types of pipe material. The roughness values in Table 1.0 for the various types of material correspond to materials in new condition.

Absolute roughness, abbreviated as $\in$ (Greek symbol epsilon), is the equivalent Nikuradse's sand-grain roughness value for the inner surface of the pipe (Nikuradse, 1950). It is a measure of the surface roughness of a material which a fluid may flow over and used in conjunction with the inside diameter to calculate the friction factor using a Moody Diagram, the Reynolds number, and the Darcy-Weisbach equation. It is dimensional and measured in units of feet (meters).

The Manning's roughness coefficient ( n ) describes the average roughness of a conduit, as determined through experimental evaluation. This coefficient is used with Manning's equation to calculate the drag the fluid will be subject to as it moves through the conduit, and the subsequent velocity of the fluid. It is empirical without a unit of measurement. The Manning roughness coefficients listed have been provided by the pipe manufacturers' trade association.

TABLE 1.0 MATERIALS FOR STORM CONDUCTORS，LEADERS，UNDERGROUND DRAIN，SEWER，AND SUBSOIL

| MATERIAL | CONDUCTOR | UNDER－ GROUND BUILDING STORM DRAIN | BUILDING STORM SEWER | STANDARD （PIPE） | STANDARD （FITTINGS） | ABSOLUTE ROUGHNESS <br> （ft） <br> $\varepsilon$ | MANNING＇S COEFFICIENT FOR ROUGHNESS n |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ABS（Schedule 40）${ }^{1,2,3}$ | X | X | X | ASTM D2661 ASTM D2680＊ | ASME A112．4．4 ASTM D2661 ASTM D2680＊ | 0.000005 | 0.009 |
| Cast－Iron ${ }^{1,2,3}$ | X | X | X | ASTM A74 ASTM A888 CISPI 301 | ASME B16．12 <br> ASTM A74 <br> ASTM A888 <br> CISPI 301 | 0.00085 | 0.012 |
| Co－Extruded ABS （Schedule 40）1，2，3 | X | X | X | ASTM F628 | ASME A112．4．4 ASTM D2661 ASTM D2680＊ | 0.000005 | 0.009 |
| Co－Extruded Composite （Schedule 40）${ }^{1}$ | X | X | X | ASTM F1488 | ASME A112．4．4 ASTM D2661 ASTM D2665 ASTM F794＊ ASTM F1866 | 0.000005 | 0.009 |
| Co－Extruded PVC $\left(\right.$ Schedule 40）${ }^{1}$ | X | X | X | ASTM F891 ASTM F1760 | ASME A112．4．4 ASTM D2665 ASTM F794＊ ASTM F1336＊ ASTM F1866 | 0.000005 | 0.009 |
| Concrete pipe ${ }^{2,3}$ |  |  | X | ASTM C14 <br> ASTM C76 <br> CSA A257．1 <br> CSA A257．2 | －－－ | 0．01－． 001 | 0.013 |
| Copper and Copper Alloys（Type DWV）1，2，3 | X | X | X | ASTM B43 <br> ASTM B75 <br> ASTM B251 <br> ASTM B302 <br> ASTM B306 | ASME B16．23 ASME B16．29 | 0.000005 | 0.011 |
| Ductile Iron ${ }^{3}$ |  |  | X | ASTM A716 ASTM A746 | AWWA C110，AWWA 153 | 0.00085 | 0.012 |
| Fiberglass ${ }^{3}$ |  |  | X | ASTM D3262 | ASTM D3840 | 0.000016 | －－－ |
| Galvanized Malleable Iron ${ }^{1,2}$ | X |  |  | － | ASME B16．3 | 0.0005 | －－－ |
| Galvanized Steel ${ }^{1,2,3}$ | X |  |  | ASTM A53 | －－ | 0.0006 | －－－ |
| Glass ${ }^{2}$ | X |  |  | ASTM C1053 | －ー | 0.000005 | －－ |
| Polyethylene ${ }^{1,2,3}$ |  | $\mathrm{X}^{2}$ | X | ASTM F714 ASTM F894 | ASTM D2949 | 0.000005 | 0．009－0．015 |
| PE ${ }^{1,2}$ |  |  | $\mathrm{X}^{2}$ | ASTM F667 | $\begin{aligned} & \text { ASTM F667/ F667M } \\ & \text { ASTM F2306/ F2306M } \\ & \text { ASTM F2763 } \\ & \text { ASTM F2947/ F2947M ² } \end{aligned}$ | 0.000005 | 0．009－0．015 |
| Polyolefin ${ }^{2}$ | X | X |  | ASTM F1412 <br> ASTM F3371 <br> CSA B181．3 | －ーー | 0.000005 | 0．009－0．015 |
| Polypropylene ${ }^{2}$ |  |  | X | ASTM F2764 <br> ASTM F2881 <br> CSA B182．13 | ASTM F2764 ASTM F2881／F2881M | 0.000005 | 0．009－0．015 |
| Polyvinylidene ${ }^{2}$ | X | X |  | ASTM F1673 CSA B181．3 | －ー－ | 0.000005 | 0．009－0．015 |

TABLE 1.0
MATERIALS FOR STORM CONDUCTORS, LEADERS, UNDERGROUND DRAIN, SEWER, AND SUBSOIL (continued)

| MATERIAL | CONDUCTOR | UNDERGROUND BUILDING STORM DRAIN | BUILDING STORM SEWER | STANDARD (PIPE) | STANDARD (FITTINGS) | ABSOLUTE ROUGHNESS <br> (ft) <br> $\varepsilon$ | MANNING'S COEFFICIENT FOR ROUGHNESS n |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PVC (Schedule 40) ${ }^{1,2,3}$ | X | X | X | ASTM D1785 ASTM D2665 ASTM F794* | ASME A112.4.4 <br> ASTM D2665 <br> ASTM F794* <br> ASTM F1866 | 0.000005 | 0.009-0.011 |
| PVC (Sewer and Drain) ${ }^{1}$ |  |  | X | ASTM D2729 | ASTM D2729 | 0.000005 | 0.009-0.011 |
| PVC PSM ${ }^{1,2,3}$ |  |  | X | ASTM D3034 | ASTM D3034 | 0.000005 | 0.009-0.011 |
| Stainless Steel 304 ${ }^{1,2,3}$ | X |  |  | ASME A112.3.1 | ASME A112.3.1 | 0.000007 | --- |
| Stainless Steel 316L ${ }^{1,2,3}$ | X | X | X | ASME A112.3.1 | ASME A112.3.1 | 0.000007 | 0.012 |
| Vitrified Clay (Extra strength) 1,2,3 |  |  | X | ASTM C700 | ASTM C700 | --- | 0.013-0.015 |

Notes:
${ }^{1}$ UPC
${ }^{2}$ IPC
${ }^{3}$ NSPC

### 2.0 Flow Capacity Equations

Flow capacity equations are used to calculate flow rates in pipe conduit based on several variables. Some of the variables are dimensional and others dimensionless. Three flow capacity equations are considered for storm drainage showing how they were derived with emphasis on the variable for roughness. Tables are provided to demonstrate how the change of variables affects the calculated flow rates between two types of pipe material having different roughness, PVC and cast iron. One flow capacity equation is for vertical drains published in NBS Monograph 31. The other two flow capacity equations for horizontal drains are Manning's equation and Darcy-Weisbach equation.

### 2.1 Vertical Storm Drain Sizing

The rational equation for flow capacities in vertical stacks is published in NBS Monograph 31 (equation 53). One of the purposes stated in the monograph was to develop the computation of loads for drainage stacks. Even though the monograph title specifies sanitary drainage, the drainage system includes the conveyance of rainwater as stated in the definitions of the monograph. Also, all testing and measurements were performed with water flow without soil or waste water. Preventing severe pressure fluctuations that could deplete fixture traps was not the only concern in the testing. The testing required that the stack loads would be carried away by gravity without creating excessive hydrostatic pressures and that the noise and vibration due to the flowing water should be reduced to a practical minimum. That the monograph was applicable for storm drainage stacks is recognized in plumbing codes and in the American Society of Plumbing Engineers (ASPE) handbook that utilizes the stack equation for sizing vertical storm drains (ASPE 2022).

The equation for flow capacity expressed in terms of the stack diameter and the water cross section is expressed in equation [1].

$$
\begin{aligned}
& q=27.8 r_{s}^{5 / 3} d^{8 / 3} \\
& \text { Where: } \\
& q=\text { volume rate of flow, gpm } \\
& r_{s}=\text { ratio of area of cross section of water stream in a drainage stack to total area of cross section of the } \\
& \quad \text { stack } \\
& d=\text { diameter of pipe, inches }
\end{aligned}
$$

Equation [1] is derived from equating the fundamental expression for velocity, equation [2], and the expression for terminal velocity, equation [3]. In equation [2], the cross-sectional area of the pipe is the cross-section of the annular layer of water determined by the ratio $r_{S}$ (see Figure 2.2).

$$
\begin{equation*}
\mathrm{V}=\frac{Q}{A_{a n}} \tag{2}
\end{equation*}
$$

Where:
$\mathrm{V}=$ velocity, $\mathrm{ft} / \mathrm{s}$
$\mathrm{Q}=$ volume rate of flow, cf/s
$\mathrm{A}_{\mathrm{an}}=$ annular cross-sectional area of the pipe, $\mathrm{ft}^{2} \frac{\pi r_{s} D^{2}}{4}$
$D=$ diameter of pipe, ft
$V_{t}=3.0\left(\frac{q}{d}\right)^{2 / 5}$
Equation [3]

Where:
$\mathrm{V}_{t}=$ terminal velocity, $\mathrm{ft} / \mathrm{s}$
$q=$ volume rate of flow, gpm
$d=$ diameter of pipe, inches
Therefore,

$$
3.0\left(\frac{q}{d}\right)^{2 / 5}=\frac{Q}{\frac{\pi r_{s} D^{2}}{4}}\left(\frac{12^{2}}{449}\right)
$$

Equation [4] equates equations [2] and [3] and adds the conversion on the right side of the equation to convert Q and $D$ in equation [2] to gallons per minute and inches respectively. Using equation [4] and solving for $Q$ yields equation [1]. In displaying these equations, note that the constant 27.8 in equation [1] is dependent on the constant in equation [3]. The constant in equation [3] is based on the roughness of cast iron pipe and therefore equation [1] reflects the flow capacity for cast iron pipe. Where the roughness differs in various types of pipe material, the constant in equation [3] will need to change. The constant in equation [3] is derived from equation [5] (Wyly, Eaton, 1961).

$$
C=2.22\left(\frac{g^{3}}{\varepsilon}\right)^{1 / 10}\left(\frac{12}{449}\right)^{2 / 5}
$$

Equation [5]

Where:
C = constant
$\mathrm{g}=$ acceleration of gravity, $32.2 \mathrm{ft} / \mathrm{s} / \mathrm{s}$
$\varepsilon=$ absolute roughness, ft
Using the absolute roughness for cast iron, 0.00085 , in equation [5] will yield the constant 3.0 found in equation [3]. Changing the absolute roughness to 0.000005 for PVC in equation [5] will yield a constant of 5.0 and therefore changing the constant in equation [3]. How this affects equation [1] will be seen as 5.0 is substituted for 3.0 in equation [4] and solving for q yields equation [6] for PVC flow capacity in stacks.

$$
q=65.2 r_{s}^{5 / 3} d^{8 / 3}
$$

Equation [6]

Using the absolute roughness values in Table 1.0 the equations in Table 2.1 can be produced. These equations can be used for any type of material having the corresponding absolute roughness.

TABLE 2.1
ABSOLUTE ROUGHNESS VALUES FOR FLOW CAPACITY EQUATION

| ABSOLUTE ROUGHNESS $(\varepsilon)$, ft. | FLOW CAPACITY EQUATION |
| :---: | :---: |
| 0.000005 | $q=65.2 r_{s}^{5 / 3} d^{8 / 3}$ |
| 0.000007 | $q=61.6 r_{s}^{5 / 3} d^{8 / 3}$ |
| 0.0005 | $q=30.3 r_{s}^{5 / 3} d^{8 / 3}$ |
| 0.0006 | $q=29.4 r_{s}^{5 / 3} d^{8 / 3}$ |
| 0.00085 | $q=27.8 r_{s}^{5 / 3} d^{8 / 3}$ |

Figure 2.1 shows the relative roughness of pipe materials using the absolute roughness values noted in the diagonal lines. This figure originated with Lewis F. Moody and was derived from the Moody chart showing the relationship of the Reynolds number to relative roughness and to the friction factor for use in the Darcy equation and applied to pipe diameters of varying material (Moody, 1944). This figure shows additional materials with their corresponding roughness coefficients.


FIGURE 2.1
RELATIVE ROUGHNESS OF PIPE MATERIAL AND FRICTION FACTORS FOR COMPLETE TURBULENCE
With respect to the $r_{s}$ value in the flow capacity equations in Table 2.1, the authors of the NBS Monograph 31 acknowledged that equation [1] was an approximate solution to the turbulent boundary layer that develops in the pipe and therefore they treated the water as if it were a rigid body sliding down the stack instead of a fluid layer with a radial velocity gradient (see Figure 2.2). The annular layer of water is considered as a rigid body moving down a plane vertical wall, acted on only by the forces of gravity and wall friction (Wyly, Eaton, 1961). The ratio of area of cross section of this annular layer of water stream in a drainage stack to total area of cross section of the stack is recommended to be no greater than one fourth to one third (Wyly, Eaton, 1961). This is to prevent the occurrences of serious pneumatic disturbances associated with excessive rates of flow causing pipe vibration and sway. The American Society of Plumbing Engineers (ASPE) base vertical drain capacities with $r_{s}$ values of $1 / 3$ (ASPE, 2022). The National Standard Plumbing Code uses the $r_{s}$ values of $7 / 24$ (a committee compromise between $1 / 3$ and 1/4). Table 2.2 shows the differences in flow capacity between PVC and cast iron using the equations in Table 2.1 and changing the $r_{s}$ value accordingly.


FIGURE 2.2
ANNULAR CROSS-SECTION OF STACK FLOWING AT DESIGN CAPACITY

TABLE 2.2
COMPARISONS OF FLOW CAPACITY IN PVC AND CAST-IRON VERTICAL STACKS

| PIPE SIZES |  |  | $\mathbf{r}_{\mathbf{s}}=\mathbf{1 / 3}$ |  | $\mathbf{r}_{\mathbf{s}}=\mathbf{7 / 2 4}$ |  | $\mathbf{r}_{\mathbf{s}}=\mathbf{1 / 4}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Nominal <br> Pipe Size | PVC Internal <br> Diameter [in] | Cast Iron <br> Internal <br> Diameter [in] | PVC Vertical <br> Flow <br> [gpm] | Cast Iron <br> Vertical Flow <br> [gpm] | PVC Vertical <br> Flow <br> [gpm] | Cast Iron <br> Vertical Flow <br> [gpm] | PVC Vertical <br> Flow <br> [gpm] | Cast Iron <br> Vertical Flow <br> [gpm] |
| 2 | 2.067 | 1.960 | 72.4 | 26.8 | 58.0 | 21.5 | 44.8 | 16.6 |
| 3 | 3.068 | 2.960 | 207.6 | 80.5 | 166.2 | 64.4 | 128.6 | 49.8 |
| 4 | 4.026 | 3.940 | 428.6 | 172.5 | 343.1 | 138.1 | 265.3 | 106.8 |
| 5 | 5.047 | 4.940 | 783.1 | 315.3 | 626.8 | 252.4 | 484.8 | 195.2 |
| 6 | 6.065 | 5.940 | 1278.2 | 515.6 | 1023.2 | 412.7 | 791.3 | 319.2 |
| 8 | 7.981 | 7.940 | 2657.9 | 1117.8 | 2127.5 | 894.8 | 1645.5 | 692.0 |
| 10 | 10.020 | 9.940 | 4875.6 | 2034.9 | 3902.8 | 1628.9 | 3018.5 | 1259.8 |
| 12 | 11.938 | 11.940 | 7777.9 | 3317.8 | 6226.0 | 2655.8 | 4815.4 | 2054.1 |
| 14 | 13.126 | --- | 10016.9 | --- | 8018.2 | --- | 6201.6 | --- |
| 15 | --- | 14.035 | --- | 5105.9 | ---- | 4087.2 | --- | 3161.1 |

Mention needs to be made that the absolute roughness $(\varepsilon)$ values in Table 1.0 for the various types of material are equivalent sand roughness (Nikuradse sand-roughness magnitude) corresponding to materials in new condition. Such material will increase in roughness with use because of corrosion, incrustation, or abrasion. Evaluating the roughness of pipe at any stage of time can only be known by making resistance measurements at different times that would fairly estimate future variation. Boundary roughness may be expected to increase with time in approximation to equation [7] (Rouse, 1978, p. 210-11).

Equation [7]

$$
\mathrm{k}=\mathrm{k}_{0}+\mathrm{at}
$$

Where:
$\mathrm{k}=$ the roughness at any time t , ft
$\mathrm{k}_{0}=$ the roughness of new material, ft
$\mathrm{a}=$ the time rate of roughness increase, $\mathrm{ft} / \mathrm{yr}$
$t=$ the time interval, number of years
However, there is no data of resistance measurements of storm drainage systems available to estimate a variation of future roughness. Further research is needed to provide this estimation. Engineers Edge displays a table showing variation from the absolute roughness in terms of mean value and recommended design value, which is an attempt to adjust the coefficient for roughness for new pipe.

Table 2.2 can be replicated for the remaining types of material in Table 1.0.

### 2.2 Horizontal Storm Drain Sizing - Manning

For horizontal storm drain sizing, the Manning's equation is the engineering choice for open channel flow. For flow to be open channel, the flow conveyance must be channeled and open to the atmosphere and not under pressure with the weight of a column of water (head pressure) pushing against the fluid under full-flow conditions. Flow in a pipe conduit (circular geometry) is analogous to open channel flow as free surface wave flow in partially filled pipes flowing under its own head (with no column of water) by the acceleration of gravity alone versus full bore conditions. Hence, the capacity of the horizontal pipe must be sufficient to convey vertical pipe discharges without creating head pressure or full bore flow.

The Manning formula is the expression of the Chezy coefficient " $c$ " in equation [8] originally developed in SI units (Manning, 1890). The Chezy coefficient was improved upon by Kutter and Manning. The Kutter formula is quite complex and introduced the " $n$ " coefficient, which were experimental values of the degree of roughness of the channel bed. Manning simplified Kutter's formula for solving the Chezy coefficient while applying the same n-coefficients as the Kutter formula and achieving practically identical results. The Manning solution to the Chezy coefficient is an explicit approximation seen in equation [9].

$$
v=c \sqrt{r_{h} s}
$$

## Equation [8]

Where:
$v=$ velocity, $\mathrm{m} / \mathrm{s}$
$c=$ coefficient, unitless
$r_{h}=$ hydraulic radius, $m$
$s=$ slope, $\mathrm{m} / \mathrm{m}$
$c=\frac{1}{n} r_{h}^{1 / 6}$
Equation [9]

Where:
c = Chezy coefficient
$r_{h}=$ hydraulic radius, $m$
$n=$ coefficient of roughness

Substituting the Manning formula in equation [9] for the Chezy coefficient " $c$ " in equation [8] yields the Manning equation in SI units seen in equation [10]. For US customary units the Manning equation is expressed in equation [11]. 1.486 is derived from the cube root of the conversion factor between meter and feet. One foot is 0.3048 meters, then $(1 / 0.3048)^{1 / 3}$ equals 1.486 . The Manning equation can also be evaluated in terms of flow rate $q$ in equation [12] in US customary units.
$v=\frac{1}{n} r_{h}^{1 / 6} r_{h}^{1 / 2} s^{1 / 2}$
$v=\frac{1}{n} r_{h}{ }^{2 / 3} s^{1 / 2}$
Equation [10]

Where:
$\mathrm{v}=$ velocity, $\mathrm{m} / \mathrm{s}$
$r_{h}=$ hydraulic radius, $m$
$s=$ slope, $\mathrm{m} / \mathrm{m}$
$n=$ coefficient of roughness

$$
V=\frac{1.486}{n} R_{h}^{2 / 3} S^{1 / 2}
$$

Where:
$\mathrm{V}=$ velocity, $\mathrm{ft} / \mathrm{s}$
$\mathrm{R}_{\mathrm{h}}=$ hydraulic radius, ft
$\mathrm{S}=$ slope, $\mathrm{ft} / \mathrm{ft}$
$\mathrm{n}=$ coefficient of roughness
$q=\frac{666.96}{n} A R_{h}^{2 / 3} S^{1 / 2}$
Equation [12]

Where:
$q$ = volume rate of flow, gpm
$\mathrm{n}=$ Manning's coefficient for roughness
$\mathrm{A}=$ cross sectional area of pipe, $\mathrm{ft}^{2}$
$\mathrm{R}_{\mathrm{h}}=$ hydraulic radius, ft
S = slope, ft/ft

When using the Mannings equation, the hydraulic radius for a circular pipe is the area of the pipe divided by the wetted perimeter seen in equation [13]. For full flowing pipes the hydraulic radius is simplified to $D / 4\left(\frac{\pi D^{2}}{4} / \pi D\right)$, where $D$ is the diameter of the pipe in feet. For partially filled pipes, the hydraulic radius is found using trigonometric functions shown in equations [14] through [17].
$\mathrm{R}_{\mathrm{h}}=\frac{A}{P}$
Where:
$\mathrm{A}=$ area, $\mathrm{ft}^{2}$
$\mathrm{P}=$ wetted perimeter, ft

If,
$\mathrm{A}=\frac{r^{2}(\Theta-\sin \theta)}{2}$
Equation [14]
$\theta=2 \cos ^{-1} \frac{r-h}{r}$
Equation [15]
$\mathrm{P}=r \Theta$
Equation [16]

Where:
$r=$ radius, ft .
$\mathrm{h}=$ depth flow y , ft.
$\theta=$ radians
Then,

$$
\begin{equation*}
\mathrm{R}_{\mathrm{h}}=\frac{\frac{r^{2}(\Theta-\sin \theta)}{2}}{r \Theta} \tag{17}
\end{equation*}
$$

Equation [17] is valid for computing the hydraulic radius in any partially flowing and full flowing pipes.

The coefficients of roughness for different types of pipe material listed in Table 1.0 have not changed much since Robert Horton's table of coefficients from 1916 (see Figure 2.3) other than the list has become more extensive with the advent of plastics and given the value similar to smooth brass and glass pipe in Horton's table (Horton, 1916). As the selection of absolute roughness for a particular pipe material is important to estimate the flow capacity for vertical pipe sizing in actual service, so also the selection of the coefficient of roughness for estimating the capacity for horizontal pipe sizing. The selection of Manning's " $n$ " should have the consideration of roughness that the pipe is expected to have after being in service for a given amount of time and not as the pipe is in its newly manufactured condition. Horton's structure of values in terms of best to bad for each type of surface material is helpful in estimating the expected condition of pipe after being in service. A modern chart of $n$-values for the Manning equation lists a range of values for only some types of material, which is most likely based on the varying manufactured products and not estimating the condition of service (see Figure 2.4).

| Surface | Best | Good | Fair | Bad |
| :---: | :---: | :---: | :---: | :---: |
| Uncoated cast-iron pip | 0.012 | 0.013 | 0.014 | 0.015 |
| Coated cast-iron pipe | 0.011 | $0.012^{*}$ | $0.013 *$ |  |
| Commercial wrought-iron pipe, black... | 0.012 | 0.013 | 0.014 | 0.015 |
| Commercial wrought-iron pipe, galvanised | 0.013 | 0.014 | 0.015 | 0.017 |
| Smooth brass and glass pipe....... | 0.009 | 0.010 | 0.011 | 0.013 |
| Smooth lockbar and welded "OD" pipe | 0.010 | $0.011{ }^{*}$ | $0.013^{\circ}$ |  |
| Riveted and spiral steel pipe. . . . . . . . . . | 0.013 | $0.015^{*}$ | 0.017* |  |
| Vitrified sewer pipe. . . . . . . . . . . . . . . . . . | $\left\{\begin{array}{l}0.010 \\ 0.011\end{array}\right\}$ | 0.013* | 0.015 | 0.017 |
| Common clay drainage | 0.011 | $0.012^{*}$ | $0.014^{*}$ | 0.017 |
| Glased brickwork. . | 0.011 | 0.012 | $0.013^{*}$ | 0.015 |
| Brick in cement mortar; brick sewers.. . | 0.012 | 0.013 | $0.015^{*}$ | 0.017 |
| Neat cement surfaces... | 0.010 | 0.011 | 0.012 | 0.013 |
| Cement mortar surfac | 0.011 | 0.012 | $0.013^{\circ}$ | 0.015 |
| Concrete pipe. | 0.012 | 0.013 | $0.015^{\circ}$ | 0.016 |
| Wood stave pipe. . . . . . . . . . . . . . . . . | 0.010 | 0.011 | 0.012 | 0.013 |

FIGURE 2.3
PARTIAL LIST OF HORTON'S VALUES OF " n " TO BE USED WITH MANNING'S FORMULA

| SURFACE MATERIAL | MANNING'S ROUGHNESS COEFFICIENT - n - |
| :---: | :---: |
| Asbestos cement | 0.011 |
| Asphalt | 0.016 |
| Brass | 0.011 |
| Brick and cement mortar sewers | 0.015 |
| Canvas | 0.012 |
| Cast or Ductile iron, new | 0.012 |
| Clay tile | 0.014 |
| Concrete - steel forms | 0.011 |
| Concrete (Cement) - finished | 0.012 |
| Concrete - wooden forms | 0.015 |
| Concrete - centrifugally spun | 0.013 |
| Copper | 0.011 |
| Corrugated metal | 0.022 |
| Earth, smooth | 0.018 |
| Earth channel - clean | 0.022 |
| Earth channel - gravelly | 0.025 |
| Earth channel - weedy | 0.030 |
| Earth channel - stony, cobbles | 0.035 |
| Floodplains - pasture, farmland | 0.035 |
| Floodplains - light brush | 0.050 |
| Floodplains - heavy brush | 0.075 |
| Floodplains - trees | 0.15 |
| Galvanized iron | 0.016 |
| Glass | 0.010 |
| Gravel, firm | 0.023 |
| Lead | 0.011 |
| Masonry | 0.025 |
| Metal - corrugated | 0.022 |
| Natural streams - clean and straight | 0.030 |
| Natural streams - major rivers | 0.035 |
| Natural streams - sluggish with deep pools | 0.040 |
| Natural channels, very poor condition | 0.060 |
| Plastic | 0.009 |
| Polyethylene PE - Corrugated with smooth inner walls | 0.009-0.015 |
| Polyethylene PE - Corrugated with corrugated inner walls | 0.018-0.025 |
| Polyvinyl Chloride PVC - with smooth inner walls | 0.009-0.011 |
| Rubble Masonry | 0.017-0.022 |
| Steel - Coal-tar enamel | 0.010 |
| Steel - smooth | 0.012 |
| Steel - New unlined | 0.011 |
| Steel - Riveted | 0.019 |
| Vitrified clay sewer pipe | 0.013-0.015 |
| Wood - planed | 0.012 |
| Wood - unplaned | 0.013 |
| Wood stave pipe, small diameter | 0.011-0.012 |
| Wood stave pipe, large diameter | 0.012-0.013 |

FIGURE 2.4
MANNING'S ROUGHNESS COEFFICIENTS (ENGINEERINGTOOLBOX.COM)

Table 2.3 was generated using equation [12], Manning's " $n$ " of 0.012 for new cast iron, 0.009 for new PVC, and both sloped at $1 / 4$ " per foot (2\%). For the Manning equation, the maximum (free surface) flow is when the depth is approximately $94 \%$ of the diameter of the pipe. At this depth, the slight loss of flow area is compensated by the decrease in wall friction and the resulting increase in flow velocity. Table 2.3 compares flow rates at three different depths. The percentages in the top row are the percentages of depth flow (h) to the interior diameter of the pipe. This table can be expanded to include other pipe materials with different Manning's " $n$ ".

TABLE 2.3
COMPARISONS OF FLOW CAPACITY IN PVC AND CAST-IRON HORIZONTAL PIPES — MANNING

| PIPE SIZES |  |  | DEPTH 50\% |  | DEPTH 75\% |  | DEPTH 94\% |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Nominal Pipe Size | PVC Internal Diameter [in] | Cast Iron Internal Diameter [in] | PVC Horizontal <br> Flow [gpm] | Cast Iron Horizontal Flow [gpm] | PVC Horizontal <br> Flow [gpm] | Cast Iron Horizontal Flow [gpm] | PVC Horizontal <br> Flow [gpm] | Cast Iron Horizontal Flow [gpm] |
| 2 | 2.067 | 1.960 | 15.3 | 9.97 | 27.9 | 18.2 | 32.9 | 21.4 |
| 3 | 3.068 | 2.960 | 43.9 | 29.9 | 80.0 | 54.6 | 94.4 | 64.4 |
| 4 | 4.026 | 3.940 | 90.6 | 64.1 | 165.2 | 117.0 | 194.9 | 138.0 |
| 5 | 5.047 | 4.940 | 165.5 | 117.2 | 301.9 | 213.8 | 356.1 | 252.2 |
| 6 | 6.065 | 5.940 | 270.2 | 191.7 | 492.7 | 349.6 | 581.3 | 412.4 |
| 8 | 7.981 | 7.940 | 561.8 | 415.6 | 1024.6 | 758.0 | 1208.7 | 894.1 |
| 10 | 10.020 | 9.940 | 1030.6 | 756.6 | 1879.5 | 1379.8 | 2217.2 | 1627.7 |
| 12 | 11.938 | 11.940 | 1644.1 | 1233.6 | 2998.4 | 2249.8 | 3537.0 | 2653.9 |
| 14 | 13.126 | ---- | 2117.4 | --- | 3861.5 | --- | 4555.2 | --- |
| 15 | --- | 15.160 | --- | 2331.8 | --- | 4252.7 | --- | 5016.6 |

### 2.3 Horizontal Storm Drain Sizing - Darcy-Weisbach

The Darcy-Weisbach equation is more commonly used for analyzing pressure pipe systems. However, an ASCE Task Force on Friction Factors in Open Channels advocated the use of the Darcy-Weisbach equation for resistance to flow in open channels (Fenton, 2010). The American Society of Plumbing Engineers includes the Darcy-Weisbach equation along with Manning and Hazen-Williams for steady, uniform flow conditions in sanitary sloping drains with caution where surging flow exists (ASPE, 2022). Dr. Roy Hunter considered Darcy-Weisbach more accurate than Kutter for pipe diameters under fourteen inches since Kutter's data was obtained from large pipes and open conduits (BH13, 1932). Hunter's table on cast iron horizontal pipe capacities was developed using the Darcy-Weisbach equation.

The Darcy-Weisbach equation is expressed in equation [18]. It can also be transformed in terms of velocity in equation [19] and flow rate in gallons per minute in equation [21].
$h_{f}=f \frac{L}{D} \frac{V^{2}}{2 g}$
Where:
$h_{f}=$ pressure drop over distance $L$
$f=$ friction coefficient
$\mathrm{L}=$ length of pipe, ft
$\mathrm{D}=$ diameter of pipe, ft
$\mathrm{V}=$ mean velocity, $\mathrm{ft} / \mathrm{s}$
$g=$ acceleration of gravity, $32.2 \mathrm{ft} / \mathrm{s} / \mathrm{s}$

Transform equation [18] to solve for velocity:
$V=\sqrt{\frac{2 g D h_{f}}{f L}}$
Substitute slope for pressure drop over distance L:
$S=\frac{h_{f}}{L}$
Where:
$\mathrm{S}=$ slope, $\mathrm{ft} / \mathrm{ft}$

To substitute hydraulic radius for diameter of pipe:
$\mathrm{R}_{\mathrm{h}}=\frac{A}{P}$
Where:
$\mathrm{R}_{\mathrm{h}}=$ hydraulic radius, ft
$\mathrm{A}=$ area, $\mathrm{ft}^{2}$
$\mathrm{P}=$ Wetted perimeter, ft

The hydraulic radius for full-flowing pipe:
$\mathrm{R}_{\mathrm{h}}=\frac{\frac{\pi D^{2}}{4}}{\pi D}=\frac{D}{4}$
and
$D=4 R_{h}$
Where:
$\mathrm{D}=$ diameter of pipe, ft
Substituting $4 R_{h}$ for $D$ and $S$ for $\frac{h_{f}}{L}$ in equation [19]:
$V=\sqrt{\frac{2 g\left(4 R_{h}\right) S}{f}}=\sqrt{\frac{8 g R_{h} S}{f}}$

Use velocity equation [20] to calculate flow rate:
$q=\frac{\pi D^{2}}{4} \times 449 \sqrt{\frac{8 g R_{h} S}{f}}$

## Equation [21]

Where:
$\mathrm{q}=$ flow rate, gpm
D = diameter, feet
$\mathrm{g}=$ acceleration of gravity, $32.2 \mathrm{ft} / \mathrm{s} / \mathrm{s}$
$\mathrm{R}_{\mathrm{h}}=$ hydraulic radius, ft
S = slope, ft/ft
$\mathrm{f}=$ friction coefficient

Substituting hydraulic radius for diameter of pipe in the Darcy-Weisbach equation is only permissible for calculating full-flowing pipe. The term $4 R_{h}$ must yield the diameter of the pipe for the substitution to be valid. Unlike the Manning equation, the Darcy-Weisbach equation does not use the hydraulic radius as one of its terms to calculate velocity. It uses the diameter of the pipe. The Darcy-Weisbach equation will not calculate velocity for partially filled pipes. $4 R_{h}$ will not express the diameter of the pipe if the hydraulic radius is less than the full area of the pipe. Therefore, this expression of the Darcy-Weisbach equation can only be used to calculate full-flowing pipe capacities.

Using equation [21] seems a simple and accurate solution knowing the diameter of pipe, hydraulic radius, and slope of pipe. However, the simplicity is deceiving once you try to calculate the friction coefficient. The ColebrookWhite equation solves $f$ for fully developed turbulent flow. It is a very cumbersome computation since it is without explicit expression and needs an iterative approach. The Moody chart shown in Figure $\mathbf{2 . 5}$ represents plots of the Colebrook equation over the range of the Reynolds number (Moody, 1944; White, 1998).


FIGURE 2.5
MOODY CHART

As the Moody chart reveals, the friction factor $f$ is dependent on the Reynolds number and the relative roughness of the pipe. The relative roughness is the ratio of absolute roughness $(\varepsilon)$ to the pipe diameter. The Moody chart is one of the most famous charts in fluid mechanics and is accurate to $\pm 15$ percent for design calculations (White, 1998). Approximations to the Colebrook-White equation having an explicit expression have become the engineering quest with no less than 33 approximations having been proposed (Zeghadnia, et al., 2019; Brkic, 2011). Out of these proposals, the Manadilli approximation was chosen in this paper to evaluate the friction factor in the Darcy-Weisbach equation (Manadilli, 1997).

The Manadilli approximation is considered simple with one step computation and having a maximum error differential of $3.31 \%$ against the entire range of the Moody chart, and a maximum error of $2.5 \%$ within the range of $5.235 \cdot 10^{3} \leq R_{e} \leq 10^{8}$ (Zeghadnia, et al., 2019). The Manadilli solution is seen in equation [22].

$$
f=\left[\frac{1}{-2 \cdot \log \left(\frac{\varepsilon}{3.7 D}+\frac{95}{R e^{0.983}}-\frac{96.82}{R e}\right)}\right]^{2}
$$

## Equation [22]

> Where:
> $f=$ friction coefficient, dimensionless
> $\varepsilon=$ absolute roughness, ft
> $\mathrm{D}=$ diameter of pipe, ft
> $\mathrm{Re}=$ Reynolds number, dimensionless

Using equation [21] and equation [22] for the friction coefficient, the Darcy-Weisbach equation yields the flow rates found in Table 2.4. Included in the table are the computed Manadilli solutions for the friction coefficient. The relative roughness ( $\varepsilon / \mathrm{D}$ ) used to compute the Manadilli formula for cast iron was $0.00085 / \mathrm{D}$, and $0.000005 / \mathrm{D}$ for PVC. The Reynolds number was also necessary to compute the Manadilli formula. The temperature of rainwater on average is between $32^{\circ} \mathrm{F}$ and $80^{\circ} \mathrm{F}$. The temperature of rainwater selected to determine the viscosity and density for the Reynolds number was $50^{\circ} \mathrm{F}$. Higher or lower temperatures will slightly influence the pipe capacity. The Reynolds number and relative roughness were computed in an MS Excel spreadsheet when calculating the flow rates in Table 2.4. When the Manadilli friction coefficients were compared to the Moody chart, they were similar to the friction factor $f$ in the Moody chart in the range of the pipe sizes used for both PVC and cast iron pipe. For the Darcy-Weisbach equation, the maximum (free surface) flow is when the depth is approximately $95 \%$ of the diameter of the pipe. At this depth, the slight loss of flow area is compensated by the decrease in wall friction and the resulting increase in flow velocity.

TABLE 2.4
COMPARISONS OF FLOW CAPACITY IN PVC AND CAST-IRON HORIZONTAL PIPES - DARCY-WEISBACH

| PIPE SIZES |  |  | FULL FLOW |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Nominal Pipe Size | PVC Internal <br> Diameter <br> [in] | Cast Iron Internal <br> Diameter <br> [in] | PVC Horizontal <br> Flow <br> [gpm] | $f$ <br> Coefficient PVC <br> (Manadilli) | Cast Iron <br> Horizontal Flow <br> [gpm] | $f$ <br> Coefficient <br> Cast Iron <br> (Manadilli) |
| 2 | 2.067 | 1.960 | 33.8 | 0.0221 | 23.8 | 0.0341 |
| 3 | 3.068 | 2.960 | 97.6 | 0.0191 | 71.8 | 0.0295 |
| 4 | 4.026 | 3.940 | 201.9 | 0.0174 | 153.7 | 0.0269 |
| 5 | 5.047 | 4.940 | 368.9 | 0.0161 | 280.2 | 0.0251 |
| 6 | 6.065 | 5.940 | 601.5 | 0.0152 | 456.6 | 0.0238 |
| 8 | 7.981 | 7.940 | 1246.8 | 0.0140 | 982.9 | 0.0219 |
| 10 | 10.020 | 9.940 | 2278.1 | 0.0130 | 1777.2 | 0.0206 |
| 12 | 11.938 | 11.940 | 3620.1 | 0.0124 | 2879.3 | 0.0196 |
| 14 | 13.126 | --- | 4651.1 | 0.0121 | ---- | ---- |
| 15 | ---- | 15.160 | --- | -- | 5392.0 | 0.0185 |

Note: Slope $=2 \% \quad(1 / 4 \mathrm{in} / \mathrm{ft})$

It is worth mentioning the friction coefficients Hunter used for the Darcy-Weisbach equation when calculating pipe capacity for cast iron drains. ${ }^{1}$ The range of coefficients were between 0.04 and 0.07 and are quite high compared to the coefficients shown in Table 2.4. Hunter clued us in why these coefficients are high by stating that the table was "based on Darcy's formula for old cast-iron pipes lined with deposit" (BH13, 1932). This again reinforces what has previously been mentioned concerning the selection of either the absolute roughness for the Darcy-Weisbach equation or Manning's " $n$ " by considering the expected condition of the pipe after being in service.

[^0]
### 3.0 Discussion

There are three significant observations when applying equation [1] for calculating vertical pipe capacity. First, the importance of using the roughness coefficient corresponding to the pipe material is evident in Table 2.2. The differences in pipe capacity with everything being equal except for the inside diameter of the pipe and the roughness coefficient, the capacities differ by 80 to 92 percent between PVC and cast iron pipe. This is significant when estimating vertical pipe capacity for different types of material.

Second, the selection of the annular ratio ( $1 / 3,7 / 24,1 / 4$ ) is also significant. Table 2.2 shows a $47 \%$ difference of flow capacity between the annular ratio of $1 / 3$ and $1 / 4$. Hunter suggested as a significant criterion for stack capacity that a stack should not flow more than $1 / 4$ to $1 / 3$ of the cross section of the stack where terminal velocity exists to prevent occurrences of severe pneumatic disturbances associated with excessive flow rates (Wyly, Eaton, 1961). Wyly stated that equation [1] for stack capacity was intended where the annular ratio is not greater than $1 / 3$ (Wyly, Eaton, 1961). Dawson and Kalinske recommended $1 / 4$ the annular ratio (Wyly, Eaton, 1961). Therefore, the flow capacities shown in the $1 / 3$ column in Table 2.2 should be considered the maximum allowable flow rates with lesser annular ratios of $7 / 24$ and $1 / 4$ as optional and would be a limiting factor for $r_{s}$ when using equation [1] for computing stack capacities. This limitation to stack flows was not only to prevent trap depletion in a sanitary system, but also to prevent noise, vibration, and pipe damage from excessive pneumatic disturbances from stacks flowing too full whether conveying soil waste or stormwater (Ballanco, 2012).

Third, the absolute roughness coefficients listed in Table 1.0 are considered for new pipe. The need for research should be considered to determine how to modify these values on an estimate of roughness after the pipe has been in service that may reduce the capacity of the pipe.

The flow rate values computed from the rational equation [1] for capacities of cast iron vertical stacks flowing full where $r_{s}=1.0$ were compared with the computed flow rate values for very rough pipes flowing full at terminal velocity based on Hunter's experiments (Wyly, Eaton, 1961). Figure 3.1 shows two computed terminal velocity curves. For a three-inch very rough pipe, terminal velocity was computed at approximately $25.5 \mathrm{ft} / \mathrm{s}$. The measured terminal velocity in three-inch cast iron pipe is shown as $32.8 \mathrm{ft} / \mathrm{s}$ (the hollow black circle). Evaluating equation [1] flowing full at terminal velocity where $r_{s}=1.0$ yields a flow rate of 520 gpm . Evaluating equation [3] at a flow rate of 520 gpm yields a terminal velocity at $23.6 \mathrm{ft} / \mathrm{s}$. This demonstrates that the rational equation [1] for stack capacities is five to ten percent less than the values for very rough pipes flowing full at terminal velocity as indicated in Figure 3.1 (Wyly, Eaton, 1961) and therefore conservative for evaluating flow capacities for storm drainage stacks.


FIGURE 3.1
TERMINAL VELOCITIES IN VERTICAL PIPES FLOWING FULL

A study that tested the flow rate capacities of numerous roof drains concluded that the head height of water ponding around the roof drain impacted the flow rate capacity of the drain and therefore recommended that the sizing of the vertical (and horizontal) storm drain pipe should be sized based on the flow rate through the roof drain (Ballanco, 2012). The vertical drain capacities published in the study were taken from the Plumbing Engineering Handbook and were determined by equation [1] although differing from the results produced in Table 2.2 because the absolute roughness was not considered for PVC pipe resulting in flow rate capacities closely calculated for cast iron pipe.

Regarding capacities for horizontal drains, Frank White cautions when using a dimensional-analysis method for the Manning equation due to the tendency to dimensional inconsistency (White, 1998). When solving for the units of " $n$ " in equation [10] in SI units, " $n$ " is not dimensionless, but the physical meaning of " $n$ " is unclear as shown in equation [23].

$$
\begin{aligned}
& \frac{m}{s}=\frac{1}{n} m^{\frac{2}{3}} x \frac{m}{}_{\frac{1}{2}}^{m} \\
& \frac{1}{n}=\frac{m^{\frac{1}{3}}}{s} \\
& n=\frac{s}{m^{\frac{1}{3}}}
\end{aligned}
$$

## Equation [23]

Equation [23] is not dimensionally homogeneous and there may be a missing scale dependence in Manning's " $n$ ". Because of this, White narrows the reliability of the Manning's equation only for water in rough channels at moderate velocities and large pipes (White, 1998). The Manning equation is useful and a reasonable solution for estimating horizontal storm drainage when large pipes are used to convey water and velocities are kept moderate. In Tables 3.1 and 3.2 Manning's equation computes velocities as high as $11 \mathrm{ft} / \mathrm{s}$ for PVC pipe and $9 \mathrm{ft} / \mathrm{s}$ for cast iron pipe. The accuracy of these velocities is dependent on Manning's " $n$ ".

The differences in flow rates between the Darcy-Weisbach equation and the Manning equation are shown in Tables 3.1 and 3.2. The Manning equation is more conservative than the Darcy-Weisbach equation in all computations. There is only an average three percent difference between Manning and Darcy-Weisbach for PVC pipe. For cast iron pipe, there is an average 9.6 percent difference between Manning and Darcy-Weisbach. The reason for the slight differences in flow rate is the calculated differences in velocity. The only variable deciding the differences in velocity between the two equations is the friction variables " $n$ " and " f ". Manning's " $n$ " remained constant for all pipe depths and pipe diameters, whereas " f " in the Manadilli approximation varied with each diameter of the pipe.

Manning's equation relates the cross-sectional area averaged velocity to the hydraulic radius and slope through the coefficient " $n$ " that only varies with surface roughness. The fact that the Manning " $n$ " is not dimensionally homogeneous reflects the empirical nature of the coefficient, making the selection of " $n$ " dependent on a picture book or a table of values provided by manufacturers, and personal judgment and experience. An ASCE Task Force on Friction Factors in Open Channels "noted that the Manning equation could be used for fully rough flow conditions, however it presented a revealing figure for the variation of resistance with Reynolds number, which showed that with Manning's equation there is continual decay of resistance with Reynolds number, even in the limit of large values, so that one could deduce that it is fundamentally flawed" (Fenton, 2010, p. 1).

The accuracy of the friction flow resistance of " $n$ " or " $f$ " in either equation needs to be better understood through experimental data from testing different pipe material. Testing flow resistance in relation to the hydraulic radius and the frictional resistance of the pipe would verify the friction coefficient proportionality to the depth of water in the pipe. This seems to be the largest discrepancy between the two friction coefficients used in the two equations. The " f " coefficients in Table 2.4 display a proportionality to the depth of water in the pipe, whereas the Manning "n" remains constant for all flow depths.

TABLE 3.1
PVC FLOW RATE COMPARISONS BETWEEN DARCY-WEISBACH AND MANNING

|  | DARCY-WEISBACH | MANNING |
| :---: | :---: | :---: |
| PIPE SIZES | FULL FLOW | DEPTH 94\% |$|$| Nominal Pipe Size | PVC Horizontal Flow <br> [gpm] | PVC Horizontal Flow <br> [gpm] |
| :---: | :---: | :---: |
| 2 | 33.8 | 32.9 |
| 3 | 97.6 | 94.4 |
| 4 | 201.9 | 194.9 |
| 5 | 368.9 | 356.1 |
| 6 | 601.5 | 581.3 |
| 8 | 1246.8 | 1208.7 |
| 10 | 2278.1 | 2217.2 |
| 12 | 3620.1 | 3537.0 |
| 14 | 4651.1 | 4555.2 |
| 15 | --- | --- |

Note: Slope $=2 \% \quad(1 / 4 \mathrm{in} / \mathrm{ft})$

TABLE 3.2
CI FLOW RATE COMPARISONS BETWEEN DARCY-WEISBACH AND MANNING

|  | DARCY-WEISBACH | MANNING |
| :---: | :---: | :---: |
| PIPE SIZES | fuLl flow | DEPTH 94\% |
| Nominal Pipe Size | PVC Horizontal Flow [gpm] | PVC Horizontal Flow [gpm] |
| 2 | 23.8 | 21.4 |
| 3 | 71.8 | 64.4 |
| 4 | 153.7 | 138.0 |
| 5 | 280.2 | 252.2 |
| 6 | 456.6 | 412.4 |
| 8 | 982.9 | 894.1 |
| 10 | 1777.2 | 1627.7 |
| 12 | 2879.3 | 2653.9 |
| 14 | --- | --- |
| 15 | 5392.0 | 5016.6 |

Note: Slope $=2 \% \quad(1 / 4 \mathrm{in} / \mathrm{ft})$

### 4.0 Conclusion

A significant criterion when using equation [1] is having the correct terminal velocity constant corresponding to the roughness coefficient of the pipe material as shown in Table 2.1. The influence of the roughness coefficient on flow capacity is significant as shown in Table 2.2. The roughness coefficients $(\varepsilon)$ shown in Table 1.0 are based on newly manufactured material. How to adjust these coefficients with consideration of roughness that the pipe is expected to have after being in service for a given amount of time is a remaining question in need of further research.

When using equation [1], the annular ratio is significant to the flow capacities in vertical pipes. The annular ratio of $1 / 3$ is the maximum ratio for pneumatic reasons with lesser annular ratios as an optional limiting factor for $r_{S}$ when using equation [1] for computing stack capacities.

Another factor to be considered for vertical pipe capacity is the flow rate through the roof drain depending on the head height of water ponding around the roof drain. Many roof drains have the potential to accelerate the flow rate as the head height of water increases. The roof drain flow rate should correspond to the capacity of the vertical pipe within its annular ratio and the capacity of the horizontal pipe to remain open channel flow.

The Manning equation [11] is the formula of choice in open channel hydraulics and has been preferred due to its simplicity without needing to further evaluate the coefficient of roughness " $n$ ". In comparison with the DarcyWeisbach equation, Manning yields conservative flow rates. The Manning equation is also more versatile for computing velocities and flow rates for partially filled pipes at various depths, whereas the Darcy-Weisbach equation only computes velocities and flow rates for full-flowing pipes. The Manning equation is not without criticism regarding its dimensional inconsistency relating to Manning's " $n$ " and the fact that " $n$ " remains constant for all pipe depths and diameters. This prompted a consideration of the Darcy-Weisbach equation for computing capacities in horizontal storm drains since the friction coefficient " f " is dependent on the Reynolds number and sensitive to flow depth and pipe diameter. The significant criterion when using equation [20] is the computation of the friction coefficient $f$ with two considerations. The first consideration is selecting the appropriate explicit approximation to the Colebrook-White equation. This report favored the Manadilli approximation because of its simplicity and accuracy. The second consideration is the selection of the roughness coefficient $(\varepsilon)$ corresponding to the material of the pipe along with the consideration of roughness the pipe is expected to have after being is service for a given time. The Manning coefficients are not dimensionally homogeneous unlike the absolute roughness ( $\varepsilon$ ) that is measured in feet or meters. The proper selection of the Manning " $n$ " or the Darcy " $f$ " to correspond to the pipe material is critical when calculating the capacity of the horizontal drain and can be better assessed by experimental data.

### 5.0 References

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[^0]:    ${ }^{1}$ Table 4 in Recommended Minimum Requirements for Plumbing (BH13, 1932) showing capacities of cast iron drains was recreated using the Darcy-Weisbach equation. Knowing the flow rates listed in the table, the friction coefficients were able to be computed.

